SYSTEM OF PARTICLES AND ROTATIONAL MOTION

FACT/DEFINITION TYPE QUESTIONS

1. The centre of mass of a body is the average position of its
   (a) mass (b) weight
   (c) either mass or weight (d) None of these
2. The centre of mass of a body
   (a) lies always outside the body
   (b) may lie within, outside on the surface of the body
   (c) lies always inside the body
   (d) lies always on the surface of the body
3. Position vector of centre of mass of two particles system is given by
   (a) $\vec{R} = \frac{m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 + m_2}$ (b) $\vec{R} = \frac{m_1\vec{r}_1 . m_2\vec{r}_2}{\vec{r}_1 + \vec{r}_2}$
   (c) $\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{\vec{r}_1 + \vec{r}_2}$ (d) $\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$
4. The motion of the centre of mass depends on
   (a) total external forces (b) total internal forces
   (c) sum of (a) and (b) (d) None of these
5. The centre of mass of a rigid body lies
   (a) inside the body (b) outside the body
   (c) neither (a) nor (b) (d) either (a) or (b)
6. The sum of moments of all the particles in a system about the centre of mass is always
   (a) maximum (b) minimum
   (c) infinite (d) zero
7. The centre of mass of two particles lies on the line
   (a) joining the particles
   (b) perpendicular to the line joining the particles
   (c) at any angle to this line
   (d) None of these
8. If the resultant of all external forces is zero, then velocity of centre of mass will be
   (a) zero (b) constant
   (c) either (a) or (b) (d) neither (a) nor (b)
9. Centre of mass of the earth and the moon system lies
   (a) closer to the earth
   (b) closer to the moon
   (c) at the mid-point of line joining the earth and the moon
   (d) cannot be predicted
10. The position of centre of mass of a system of particles does not depend upon the
    (a) mass of particles
    (b) symmetry of the body
    (c) position of the particles
    (d) relative distance between the particles
11. In rotatory motion, linear velocities of all the particles of the body are
    (a) same (b) different
    (c) zero (d) cannot say
12. Which of the following is invalid equation? (where τ, ω, L and α have their usual meanings)
    (a) $\vec{\tau} = I\vec{\omega}$ (b) $\vec{L} = I\vec{\omega}$
    (c) $\vec{\tau} = I\vec{\alpha}$ (d) All of these
13. The time rate of change of angular momentum of a particle is equal to
    (a) force (b) acceleration
    (c) torque (d) linear momentum
14. Which component of force contributes to the torque?
    (a) Radial component
    (b) Transverse component
    (c) Both (a) and (b)
    (d) Either radial or transverse
15. The wide handle of screw is based upon
    (a) Newton's second law of motion
    (b) law of conservation of linear momentum
    (c) turning moment of force
    (d) None of these
16. Which of the following is an expression for power?
    (a) $P = \tau\omega$ (b) $P = I\alpha$
    (c) $P = I\omega$ (d) $P = \tau\alpha$
17. Which of the following statements about angular momentum is correct?
    (a) It is directly proportional to moment of inertia
    (b) It is a scalar quantity
    (c) both (a) and (b)
    (d) None of these
18. A couple produces
    (a) linear motion (b) rotational motion
    (c) both (a) and (b) (d) neither (a) nor (b)

19. According to the principle of conservation of angular momentum, if moment of inertia of a rotating body decreases, then its angular velocity
(a) decreases (b) increases
(c) remains constant (d) becomes zero

20. When a mass is rotating in a plane about a fixed point, its angular momentum is directed along the
(a) radius of orbit
(b) tangent to the orbit
(c) line parallel to plane of rotation
(d) line perpendicular to plane of rotation

21. The motion of a rigid body which is not pivoted or fixed in some way is either a pure ...*A*... or a combination of translation and rotation. The motion of a rigid body which is pivoted or fixed in some way is ...*B*...
Here, *A* and *B* refer to
(a) rotation and translation
(b) translation and rotation
(c) translation and the combination of rotation and translation
(d) None of the above

22. The moment of inertia of a ...*A*... body about an axis ...*B*... to its plane is equal to the sum of its moments of inertia about two ...*C*... axes concurrent with perpendicular axis and lying in the plane of the body.
Here, *A*, *B* and *C* refer to
(a) three dimensional, perpendicular and perpendicular
(b) planar, perpendicular and parallel
(c) planar, perpendicular and perpendicular
(d) three dimensional, parallel and perpendicular

23. During summersault, a swimmer bends his body to
(a) increase moment of Inertia
(b) decrease moment of Inertia
(c) decrease the angular momentum
(d) reduce the angular velocity

24. The moment of inertia of a uniform circular disc of radius 'R' and mass 'M' about an axis passing from the edge of the disc and normal to the disc is
(a) MR^2 (b) $\frac{1}{2}MR^2$
(c) $\frac{3}{2}MR^2$ (d) $\frac{7}{2}MR^2$

25. Rotational analogue of force in linear motion is
(a) weight (b) angular momentum
(c) moment of inertia (d) torque

26. A boy comes and sits suddenly on a circular rotating table. What will remain conserved for the table-boy system?
(a) Angular velocity (b) Angular momentum
(c) Linear momentum (d) Angular acceleration

27. In rotation of a rigid body about a fixed axis, every ...*A*...of the body moves in a ...*B*..., which lies in a plane ...*C*... to the axis and has its centre on the axis.
Here, *A*, *B* and *C* refer to
(a) particle, perpendicular and circle
(b) circle, particle and perpendicular
(c) particle, circle and perpendicular
(d) particle perpendicular and perpendicular

28. Moment of inertia does not depend upon
(a) distribution of mass
(b) axis of rotation
(c) point of application of force
(d) None of these

29. Force change the ...*X*... state of motion of a rigid body.
(a) Here X refers to rotational
(b) translational
(c) rotational and translational in general
(d) None of the above

30. Moment of inertia of a circular wire of mass *M* and radius *R* about its diameter is
(a) $MR^2/2$ (b) MR^2
(c) $2MR^2$ (d) $MR^2/4$.

31. Moment of inertia does not depend upon
(a) angular velocity of body
(b) shape and size
(c) mass
(d) position of axis of rotation

32. Moment of inertia of a hollow cylinder of mass M and radius r about its own axis is
(a) $\frac{2}{3}Mr^2$ (b) $\frac{2}{5}Mr^2$
(c) $\frac{1}{3}Mr^2$ (d) Mr^2

33. Which of the following has the highest moment of inertia when each of them has the same mass and the same outer radius
(a) a ring about its axis, perpendicular to the plane of the ring
(b) a disc about its axis, perpendicular to the plane of the ring
(c) a solid sphere about one of its diameters
(d) a spherical shell about one of its diameters

34. Radius of gyration of a body depends upon
(a) axis of rotation (b) translational motion
(c) shape of the body (d) area of the body

35. The correct relation between moment of inertia I, radius of gyration k and mass M of the body is
(a) $K = I^2M$ (b) $K = IM^2$
(c) $K = \sqrt{\frac{M}{I}}$ (d) $K = \sqrt{\frac{I}{M}}$

36. Choose the wrong statement from the following.
(a) The centre of mass of a uniform circular ring is at its geometric centre
(b) Moment of inertia is a scalar quantity
(c) Radius of gyration is a vector quantity
(d) For same mass and radius, the moment of inertia of a ring is twice that of a uniform disc

37. If two circular discs A and B are of same mass but of radii r and 2r respectively, then the moment of inertia of A is
(a) the same as that of B (b) twice that of B
(c) four times that of B (d) 1/4 that of B

38. Analogue of mass in rotational motion is
(a) moment of inertia (b) angular momentum
(c) torque (d) None of these

39. For a given mass and size, moment of inertia of a solid disc is
(a) more than that of a ring
(b) less than that of a ring
(c) equal to that of a ring
(d) depend on the material of ring and disc

40. What is the moment of inertia of a solid sphere about its diameter?
(a) $\frac{2}{5}MR^2$ (b) $\frac{1}{5}MR^2$
(c) $\frac{2}{3}MR^2$ (d) MR^2

41. If I_{xy} is the moment of inertia of a ring about a tangent in the plane of the ring and $I_{x'y'}$ is the moment of inertia of a ring about a tangent perpendicular to the plane of the ring then
(a) $I_{xy} = I_{x'y'}$ (b) $I_{xy} = \frac{1}{2}I_{x'y'}$
(c) $I_{x'y'} = \frac{3}{4}I_{xy}$ (d) $I_{xy} = \frac{3}{4}I_{x'y'}$

42. Moment of inertia of a rigid body depends on
(a) Mass of the body (b) Shape of the body
(c) Size of the body (d) All of these

43. Which of the following is not the moment of inertia of a uniform circular disc along any axis?
(a) $\frac{1}{2}MR^2$ (b) $\frac{3}{2}MR^2$
(c) $\frac{1}{4}MR^2$ (d) $\frac{3}{4}MR^2$

44. Which of the following is/are essential condition for mechanical equilibrium of a body?
(a) Total force on the body should be zero
(b) Total torque on the body should be zero
(c) Both (a) and (b)
(d) Total linear momentum should be zero

45. Which of the following is incorrect?
(a) $\vec{v} = \vec{\omega} \times \vec{r}$ (b) $\vec{\tau} = \vec{F} \times \vec{r}$
(c) $\vec{L} = \vec{r} \times \vec{p}$ (d) None of these

46. Which of the following is not an expression for kinetic energy?
(a) $k = \frac{1}{2}MR^2\omega^2$ (b) $k = \frac{1}{2}I\omega^2$
(c) $k = \frac{1}{2}mv^2$ (d) None of these

47. A particle moving in a circular path has an angular momentum of L. If the frequency of rotation is halved, then its angular momentum becomes
(a) $\frac{L}{2}$ (b) L
(c) $\frac{L}{3}$ (d) $\frac{L}{4}$

48. A circular disc A and a ring B have same mass and same radius. If they are rotated with the same angular speed about their own axis, then
(a) A has less moment of inertia than B
(b) A has less rotational kinetic energy than B
(c) A and B have the same angular momentum
(d) A has greater angular momentum than B

49. The angular momentum of a system of particle is conserved
(a) when no external force acts upon the system
(b) when no external torque acts upon the system
(c) when no external impulse acts upon the system
(d) when axis of rotation remains same

50. Angular momentum of the particle rotating with a central force is constant due to
(a) constant torque
(b) constant force
(c) constant linear momentum
(d) zero torque

51. When a body starts to roll on an inclined plane, its potential energy is converted into
(a) translation kinetic energy only
(b) translation and rotational kinetic energy
(c) rotational energy only
(d) None of these

52. A body cannot roll without slipping on a
(a) rough horizontal surface
(b) smooth horizontal surface
(c) rough inclined surface
(d) smooth inclined surface

53. A fan is moving around its axis, What will be its motion regarded as ?
(a) Pure rolling (b) Rolling with shipping
(c) Skidding (d) Pure rotation

54. A sphere rolls on a rough horizontal surface and stops. What does the force of friction do?
(a) It decreases the angular velocity
(b) It decreases the linear velocity
(c) It increases the angular velocity
(d) It decreases both angular and linear velocity

55. If a ring, a solid cylinder and a solid sphere roll down the same inclined plane without slipping then
(a) ring has the least velocity of centre of mass at the bottom of inclined plane
(b) sphere has the least velocity of centre of mass at the bottom of the inclined plane
(c) cylinder has the greatest velocity of centre of mass at the bottom of the inclined plane
(d) sphere has the least velocity of centre of mass at the bottom of the inclined plane.

STATEMENT TYPE QUESTIONS

56. Consider the following statements and select the correct statements.

I. The position of centre of mass depends upon the shape of the body

II. The position of centre of mass depends upon the distribution of mass

III. The positon of centre of mass is independent of the co-ordinate system chosen

(a) I and II only (b) II and III only
(c) I and III only (d) I, II and III

57. Which of the following statements are incorrect about centre of mass?

I. Centre of mass can coincide with geometrical centre of a body

II. Centre of mass of a system of two particles does not always lie on the line joining the particles

III. Centre of mass should always lie on the body.

(a) II and III (b) I and II
(c) I and III (d) I, II and III

58. Consider the following statements and choose the correct option.

I. Position vector of centre of mass of two particles of equal mass is equal to the position vector of either particle.

II. Centre of mass is always at the mid-point of the line joining two particles

III. Centre of mass of a body can lie where there is no mass

(a) I and II (b) II only
(c) III only (d) I, II and III

59. Consider the following statements and select the correct statement(s).

I. Angular velocity is a scalar quantity

II. Linear velocity is a vector quantity

III. About a fixed axis, angular velocity has fixed direction

IV. Every point on a rigid rotating body has different angular velocity

(a) I only (b) II only
(c) II and III (d) III and IV

60. Consider the following statements and select the correct option.

I. Moment of a couple depends on the point about which moment is taken.

II. Principle of moments holds only when parallel forces F_1 and F_2 are perpendicular to the lever

III. Centre of mass depends on the gravity

IV. Centre of mass depends on the distribution of mass of the body

(a) I and II (b) III and IV
(c) I, II and III (d) IV only

61. Consider the following statements and choose the incorrect statement(s).

I. A body in translatory motion cannot have angular momentum

II. If $\vec{A}$ points vertically upwards and $\vec{B}$ points towards east then, $\vec{A} \times \vec{B}$ points along South

(a) I only (b) II only
(c) I and II (d) None of these

62. Select the correct statement(s) from the following.

I. Moment of inertia is a scalar quantity

II. Angular acceleration requires torque

III. The rotational equivalent of distance is radius

IV. State of rest or motion of centre of mass can never be changed

(a) I only (b) II only
(c) I and II (d) II and IV

63. Consider the following statements and select the correct statement(s).

I. Two satellites of equal masses orbiting the earth at different heights have equal moments of inertia

II. If earth were to shrink suddenly, length of the day will increase

III. Centre of gravity cannot coincide with centre of mass

(a) I only (b) II only
(c) I and II (d) I, II and III

MATCHING TYPE QUESTIONS

64. Match Column I and Column II

Column I	Column II
(A) Moment of inertia	(1) Twice the product of mass and areal velocity of the particle
(B) Radius of gyration	(2) The product of masses of the various particles and square of their perpendicular distances
(C) Angular momentum	(3) The root mean square distance of the particles from the axis of rotation
(D) Torque	(4) The product of force and its perpendicular distance

(a) (A)→(2); (B)→(3); C→(1); (D)→(4)
(b) (A)→(1); (B)→(2); C→(4); (D)→(3)
(c) (A)→(2); (B)→(1); C→(4); (D)→(3)
(d) (A)→(2); (B)→(4); C→(1); (D)→(3)

65.

Column I	Column II
(A) Rolling motion	(1) Torque
(B) Rate of change of angular momentum	(2) Rotatory motion
(C) Hollow cylinder about axis	(3) $I_z + Ma^2$
(D) Theorem of parallel axes	(4) MR^2

(a) (A)→(1); (B)→(3); C→(4); (D)→(2)
(b) (A)→(3); (B)→(2); C→(4); (D)→(1)
(c) (A)→(2); (B)→(1); C→(4); (D)→(3)
(d) (A)→(3); (B)→(1); C→(2); (D)→(4)

66.

Column I	Column II
(A) Translational equilibrium	(1) $\Sigma F = 0$
(B) Moment of inertia of disc	(2) MR^2
(C) Rotational equilibrium	(3) $\frac{1}{2}I\omega^2$
(D) Kinetic energy of rolling body	(4) $\frac{1}{2}mV_{cm}^2 + \frac{1}{2}I\omega^2$
	(5) $\Sigma\tau = 0$
(E) Moment of inertia of ring	(6) $MR^2/2$

(a) (A)→(1); (B)→(6); C→(5); (D)→(4); (E)→(2)
(b) (A)→(4); (B)→(3); C→(2); (D)→(1); (E)→(6)
(c) (A)→(6); (B)→(5); C→(3); (D)→(4); (E)→(2)
(d) (A)→(1); (B)→(2); C→(4); (D)→(5); (E)→(6)

67. Match Column I and Column II.

Column I (*Quantity*)	Column II (*Expression*)
(A) Angular momentum	(1) $\vec{r} \times (m\vec{v})$
(B) Impulse	(2) $\frac{1}{2}I\omega^2$
(C) Torque	(3) $\vec{r} \times \vec{F}$
(D) Rotational energy	(4) $m\Delta\vec{v}$

(a) (A)→(4); (B)→(2); C→(1); (D)→(3)
(b) (A)→(1); (B)→(2); C→(4); (D)→(3)
(c) (A)→(2,4); (B)→(1); C→(5); (D)→(3)
(d) (A)→(1); (B)→(4); C→(3); (D)→(2)

68. Match Column I and Column II.

Column I	Column II
(A) Moment of Inertia of a solid uniform sphere about the diameter	(1) MR^2
(B) Moment of inertia of a thin uniform spherical shell about the tangent	(2) $\frac{1}{2}MR^2$
(C) Moment of inertia of a uniform disc through centre of a mass and perpendicular to plane of the disc	(3) $\frac{5}{3}MR^2$
(D) Moment of inertia of disc about tangent in the plane of disc.	(4) $\frac{2}{5}MR^2$
	(5) $\frac{5}{4}MR^2$

(a) (A)→(3,2); (B)→(3); C→(5); (D)→(4)
(b) (A)→(5); (B)→(2); C→(1); (D)→(3)
(c) (A)→(4); (B)→(3); C→(2); (D)→(5)
(d) (A)→(4); (B)→(5); C→(2); (D)→(1)

69. Match Column I (Body rolling on a surface without slipping) with Column II (Ratio of Translational energy to Rotational energy.

Column I	Column II
(A) Circular ring	(1) 1/2
(B) Circular disc	(2) 1
(C) Solid sphere	(3) 3/2
(D) Spherical shell	(4) 2
	(5) 5/2

(a) (A)→(4); (B)→(2); C→(1); (D)→(3)
(b) (A)→(1); (B)→(2); C→(4); (D)→(3)
(c) (A)→(2); (B)→(1); C→(5); (D)→(3)
(d) (A)→(2); (B)→(4); C→(5); (D)→(3)

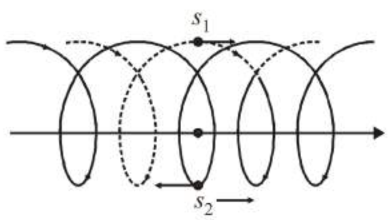
70. A rigid body of mass M and radius R rolls without slipping on an inclined plane of inclination θ, under gravity. Match the type of body Column I with magnitude of the force of friction Column II

Column I	Column II
(A) For ring	(1) $\frac{Mg\sin\theta}{2.5}$
(B) For solid sphere	(2) $\frac{Mg\sin\theta}{3}$
(C) For solid cylinder	(3) $\frac{Mg\sin\theta}{3.5}$
(D) For hollow spherical shell	(4) $\frac{Mg\sin\theta}{2}$

(a) (A)→(4); (B)→(3); C→(2); (D)→(1)
(b) (A)→(1); (B)→(2); C→(4); (D)→(3)
(c) (A)→(2); (B)→(1); C→(5); (D)→(3)
(d) (A)→(2); (B)→(4); C→(1); (D)→(3)

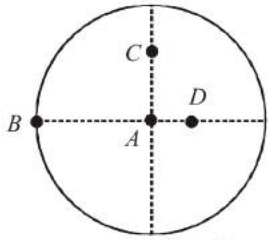
DIAGRAM TYPE QUESTIONS

71. The motion of binary stars, S_1 and S_2 is the combination of ....*X*.... and ....*Y*..... Here, *X* and *Y* refer to



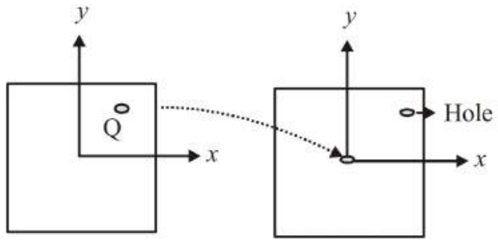
(a) motion of the CM and motion about the CM
(b) motion about the CM and motion of one star
(c) position of the CM and motion of the CM
(d) motion about CM and position of one star

72. The moment of inertia of a uniform circular disc (figure) is maximum about an axis perpendicular to the disc and passing through



(a) B (b) C
(c) D (d) A

73. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind. Then the moment of inertia about the z-axis



(a) increases
(b) decreases
(c) remains same
(d) changed in unpredicted manner.

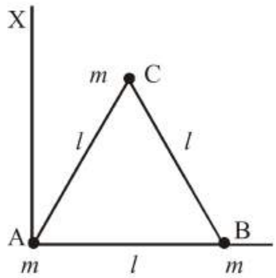
74. Three particles, each of mass m gram, are situated at the vertices of an equilateral triangle ABC of side ℓ cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC, in gram-cm^2 units will be

(a) $\frac{3}{2}m\ell^2$

(b) $\frac{3}{4}m\ell^2$

(c) $2\,m\ell^2$

(d) $\frac{5}{4}m\ell^2$



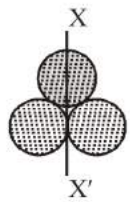
75. Three identical spherical shells, each of mass m and radius r are placed as shown in figure. Consider an axis XX' which is touching to two shells and passing through diameter of third shell. Moment of inertia of the system consisting of these three spherical shells about XX' axis is

(a) $3mr^2$

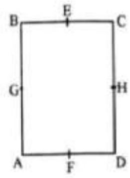
(b) $\frac{16}{5}mr^2$

(c) $4mr^2$

(d) $\frac{11}{5}mr^2$



76. The moment of inertia of the rectangular plate ABCD, (AB = 2 BC) is minimum along the axis



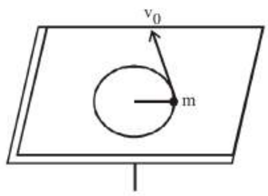
(a) GH
(b) EF
(c) BC
(d) AC

77. A rod PQ of mass M and length L is hinged at end P. The rod is kept horizontal by a massless string tied to point Q as shown in figure. When string is cut, the initial angular acceleration of the rod is

(a) g/L (b) $2g/L$

(c) $\frac{2g}{3L}$ (d) $\frac{3g}{2L}$

78. A mass m moves in a circle on a smooth horizontal plane with velocity v_0 at a radius R_0. The mass is attached to string which passes through a smooth hole in the plane as shown.

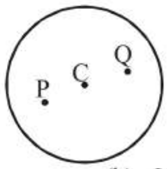


The tension in the string is increased gradually and finally mass m moves in a circle of radius $\frac{R_0}{2}$. The final value of the kinetic energy is

(a) $\frac{1}{4}mv_0^2$ (b) $2mv_0^2$

(c) $\frac{1}{2}mv_0^2$ (d) mv_0^2

79. A disc is rolling (without slipping) on a horizontal surface C is its centre and Q and P are two points equidistant from C. Let V_p, V_q and V_c be the magnitude of velocities of points P, Q and C respectively, then



(a) $V_Q > V_C > V_P$ (b) $V_Q < V_C < V_P$

(c) $V_Q = V_P,\ V_C = \frac{1}{2}V_P$ (d) $V_Q < V_C > V_P$

ASSERTION- REASON TYPE QUESTIONS

Directions : Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

80. **Assertion:** Centre of mass of a ring lies at its geometric centre though there is no mass.
Reason: Centre of mass is independent of mass.

81. **Assertion :** When you lean behind over the hind legs of the chair, the chair falls back after a certain angle.
Reason : Centre of mass lying outside the system makes the system unstable.

82. **Assertion :** The centre of mass of a two particle system lies on the line joining the two particle, being closer to the heavier particle.
Reason : Product of mass of particle and its distance from centre of mass is numerically equal to product of mass of other particle and its distance from centre of mass.

83. **Assertion :** The centre of mass of system of *n* particles is the weighted average of the position vector of the n particles making up the system.
Reason : The position of the centre of mass of a system in independent of coordinate system.

84. **Assertion :** The centre of mass of an isolated system has a constant velocity.
Reason : If centre of mass of an isolated system is already at rest, it remains at rest.

85. **Assertion :** The centre of mass of a body may lie where there is no mass.
Reason : Centre of mass of body is a point, where the whole mass of the body is supposed to be concentrated.

86. **Assertion :** The position of centre of mass of body depend upon shape and size of the body.
Reason : Centre of mass of a body lies always at the centre of the body

87. **Assertion :** If no external force acts on a system of particles, then the centre of mass will not move in any direction.
Reason : If net external force is zero, then the linear momentum of the system remains constant.

88. **Assertion :** A particle is moving on a straight line with a uniform velocity, its angular momentum is always zero.
Reason : The momentum is not zero when particle moves with a uniform velocity.

89. **Assertion :** The earth is slowing down and as a result the moon is coming nearer to it.
Reason : The angular momentum of the earth moon system is conserved.

90. **Assertion :** For a system of particles under central force field, the total angular momentum is conserved.
Reason : The torque acting on such a system is zero.

91. **Assertion :** It is harder to open and shut the door if we apply force near the hinge.
Reason : Torque is maximum at hinge of the door.

92. **Assertion :** Torque is equal to rate of change of angular momentum.
Reason : Angular momentum depends on moment of inertia and angular velocity.

93. **Assertion:** When no external torque acts on a body, its angular velocity remains constant as long as moment of inertia is constant.

Reason: Torque $\tau = 0$; $\frac{dL}{dt} = 0$, L = constant.

$L = I\omega = \text{constant}$

94. **Assertion:** Torque is a vector quantity directed opposite to the applied force.

Reason: Torque $\vec{\tau} = -\vec{r} \times \vec{F}$

95. **Assertion:** When axis of rotation passes through the centre of gravity, then the moment of inertia of a rigid body increases.
Reason: At the centre of gravity mass gets concentrated and moment of inertia increases.

96. **Assertion:** An ice-skater stretches out arms-legs during performance.
Reason: Stretching out arms-legs helps the performer to balance his or her body so that he or she does not fall.

97. **Assertion :** If polar ice melts, days will be longer.
Reason : Moment of inertia decreases and thus angular velocity increases.

98. **Assertion :** Moment of inertia of a particle is same, whatever be the axis of rotation
Reason : Moment of inertia depends on mass and distance of the particles.

99. **Assertion :** Radius of gyration of body is a constant quantity.
Reason : The radius of gyration of a body about an axis of rotation may be defined as the root mean square distance of the particle from the axis of rotation.

100. **Assertion:** A rigid disc rolls without slipping on a fixed rough horizontal surface with uniform angular velocity. Then the acceleration of lowest point on the disc is zero.

Reason : For a rigid disc rolling without slipping on a fixed rough horizontal surface, the velocity of the lowest point on the disc is always zero.

101. **Assertion :** A wheel moving down a frictionless inclined plane will slip and not roll on the plane.
Reason : It is the frictional force which provides a torque necessary for a body to roll on a surface.

102. **Assertion :** When a sphere is rolls on a horizontal table it slows down and eventually stops.
Reason : When the sphere rolls on the table, both the sphere and the surface deform near the contact. As a result, the normal force does not pass through the centre and provide an angular declaration.

103. **Assertion :** The velocity of a body at the bottom of an inclined plane of given height, is more when it slides down the plane, compared to when it is rolling down the same plane.

Reason : In rolling, down, a body acquires both, kinetic energy of translation and rotation.

104. **Assertion :** The total kinetic energy of a rolling solid sphere is the sum of translational and rotational kinetic energies.

Reason : For all solid bodies total kinetic energy is always twice the translational kinetic energy.

CRITICALTHINKING TYPE QUESTIONS

105. Two particles of mass m_1 and m_2 $(m_1 > m_2)$ attract each other with a force inversely proportional to the square of the distance between them. If the particles are initially held at rest and then released, the centre of mass will

(a) move towards m_1 (b) move towards m_2
(c) remains at rest (d) None of these

106. A shell following a parabolic path explodes somewhere in its flight. The centre of mass of fragments will continue to move in

(a) vertical direction (b) any direction
(c) horizontal direction (d) same parabolic path

107. A man stands at one end of a boat which is stationary in water. Neglect water resistance. The man now moves to the other end of the boat and again becomes stationary. The centre of mass of the 'man plus boat' system will remain stationary with respect to water

(a) in all cases
(b) only when the man is stationary initially and finally
(c) only if the man moves without acceleration on the boat
(d) only if the man and the boat have equal masses

108. There are some passengers inside a stationary railway compartment. The centre of mass of the compartment itself (without the passengers) is C_1, while the centre of mass of the 'compartment plus passengers' system is C_2. If the passengers move about inside the compartment then

(a) both C_1 and C_2 will move with respect to the ground
(b) neither C_1 nor C_2 will be stationary with respect to the ground
(c) C_1 will move but C_2 will be stationary with respect to the ground
(d) C_2 will move but C_1 will be stationary with respect to the ground

109. A stick is thrown in the air and lands on the ground at some distance from the thrower. The centre of mass of the stick will move along a parabolic path

(a) in all cases
(b) only if the stick is uniform
(c) only if the stick has linear motion but no rotational motion
(d) only if the stick has a shape such that its centre of mass is located at some point on it and not outside it

110. Consider a system of two particles having masses m_1 and m_2. If the particle of mass m_1 is pushed towards the centre of mass particles through a distance d, by what distance would the particle of mass m_2 move so as to keep the mass centre of particles at the original position?

(a) $\frac{m_2}{m_1}d$ (b) $\frac{m_1}{m_1+m_2}d$
(c) $\frac{m_1}{m_2}d$ (d) d

111. Three masses are placed on the x-axis : 300 g at origin, 500 g at $x = 40$ cm and 400 g at $x = 70$ cm. The distance of the centre of mass from the origin is

(a) 40 cm (b) 45 cm
(c) 50 cm (d) 30 cm

112. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}$ M and a body C of mass $\frac{2}{3}$ M. The centre of mass of bodies B and C taken together shifts compared to that of body A

(a) does not shift
(b) depends on height of breaking
(c) towards body B
(d) towards body C

113. A circular disc of radius R is removed from a bigger circular disc of radius 2R such that the circumferences of the discs coincide. The centre of mass of the new disc is α / R form the centre of the bigger disc. The value of α is

(a) 1/4 (b) 1/3
(c) 1/2 (d) 1/6

114. The centre of mass of three bodies each of mass 1 kg located at the points (0, 0), (3, 0) and (0, 4) in the XY plane is

(a) $\left(\frac{4}{3}, 1\right)$ (b) $\left(\frac{1}{3}, \frac{2}{3}\right)$
(c) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (d) $\left(1, \frac{4}{3}\right)$

115. The instantaneous angular position of a point on a rotating wheel is given by the equation $\theta(t) = 2t^3 - 6t^2$. The torque on the wheel becomes zero at

(a) t = 1s (b) t = 0.5 s
(c) t = 0.25 s (d) t = 2s

116. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both ends. The tube is then rotated in a horizontal plane about one of its ends with uniform angular speed ω. What is the force exerted by the liquid at the other end?

(a) $\frac{ML\omega^2}{2}$ (b) $ML\omega^2$
(c) $\frac{ML\omega^2}{4}$ (d) $\frac{ML\omega^2}{8}$

117. A planet is moving around the sun in an elliptical orbit. Its speed is
(a) same at all points of the orbit
(b) maximum when it is nearest to sun
(c) maximum when it is farthest from the sun
(d) cannot say

118. A man standing on a rotating table is holding two masses at arm's length. Without moving his arms, he drops the two masses. His angular speed will
(a) increase (b) decrease
(d) become zero (d) remain constant

119. When sand is poured on a rotating disc, its angular velocity will
(a) decrease (b) increase
(c) remain constant (d) None of the above

120. A disc is given a linear velocity on a rough horizontal surface then its angular momentum is
(a) conserved about COM only
(b) conserved about the point of contact only
(c) conserved about all the points
(d) not conserved about any point.

121. Standing on a turn table, you are rotating holding weights in your hands outstretched horizontally, If you suddenly draw your hands and weights towards your chest, you will now
(a) stop rotating (b) rotate as before
(c) rotate slower (d) rotate faster

122. Of the two eggs which have identical sizes, shapes and weights, one is raw, and other is half boiled. The ratio between the moment of inertia of the raw to the half boiled egg about central axis is
(a) one (b) greater than one
(c) less than one (d) not comparable

123. A gymnast takes turns with her arms and legs stretched. When she pulls her arms and legs in
(a) the angular velocity decreases
(b) the moment of inertia decreases
(c) the angular velocity stays constant
(d) the angular momentum increases

124. One solid sphere A and another hollow sphere B are of same mass and same outer radii, Their moments of inertia about their diameters are respectively I_A and I_B, such that
(a) $I_A = I_B$ (b) $I_A > I_B$
(c) $I_A < I_B$ (d) $I_A / I_B = \rho_A = \rho_B$
Here ρ_A and ρ_B represent their densities.

125. A diver in a swimming pool bends his head before diving. It
(a) increases his linear velocity
(b) decreases his angular velocity
(c) increases his moment of inertia
(d) decreases his moment of inertia

126. A ring of mass m and radius r is melted and then moulded into a sphere. The moment of inertia of the sphere will be
(a) more than that of the ring
(b) less than that of the ring
(c) equal to that of the ring
(d) None of these

127. A wheel having moment of inertia 2 kg-m^2 about its vertical axis, rotates at the rate of 60 rpm about this axis, The torque which can stop the wheel's rotation in one minute would be
(a) $\frac{\pi}{18}$ Nm (b) $\frac{2\pi}{15}$ Nm
(c) $\frac{\pi}{12}$ Nm (d) $\frac{\pi}{15}$ Nm

128. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is
(a) $\frac{(I_1+I_2)\omega}{I_1}$ (b) $\frac{I_2\omega}{I_1+I_2}$
(c) ω (d) $\frac{I_1\omega}{I_1+I_2}$

129. The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius about a tangential axis in the plane of the ring is
(a) $1:\sqrt{2}$ (b) $1:3$
(c) $2:1$ (d) $\sqrt{5}:\sqrt{6}$

130. Two bodies have their moments of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio
(a) $2:1$ (b) $1:2$
(c) $\sqrt{2}:1$ (d) $1:\sqrt{2}$

131. The moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its midpoint and perpendicular to its length is I_0. Its moment of inertia about an axis passing through one of its ends and perpendicular to its length is
(a) $I_0 + ML^2/2$ (b) $I_0 + ML^2/4$
(c) $I_0 + 2ML^2$ (d) $I_0 + ML^2$

132. Four point masses, each of value m, are placed at the corners of a square ABCD of side ℓ. The moment of inertia of this system about an axis passing through A and parallel to BD is

(a) $2m\ell^2$ (b) $\sqrt{3}m\ell^2$

(c) $3m\ell^2$ (d) $m\ell^2$

133. Consider a uniform square plate of side 'a' and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(a) $\frac{5}{6}ma^2$ (b) $\frac{1}{12}ma^2$

(c) $\frac{7}{12}ma^2$ (d) $\frac{2}{3}ma^2$

134. Point masses 1, 2, 3 and 4 kg are lying at the points (0, 0, 0), (2, 0, 0), (0, 3, 0) and (–2, –2, 0) respectively. The moment of inertia of this system about X-axis will be

(a) 43 kg m^2 (b) 34 kg m^2

(c) 27 kg m^2 (d) 72 kg m^2

135. The moment of inertia of a circular disc of mass M and radius R about an axis passing through the centre of mass is I_0. The moment of inertia of another circular disc of same mass and thickness but half the density about the same axis is

(a) $\frac{I_0}{8}$ (b) $\frac{I_0}{4}$

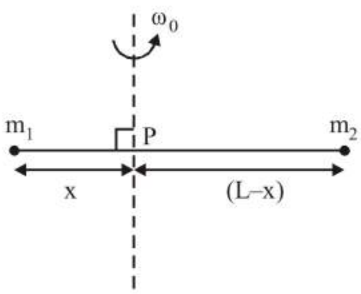
(c) $8I_0$ (d) $2I_0$

136. An automobile moves on a road with a speed of 54 km h^{-1}. The radius of its wheels is 0.45 m and the moment of inertia of the wheel about its axis of rotation is 3 kg m^2. If the vehicle is brought to rest in 15s, the magnitude of average torque transmitted by its brakes to the wheel is :

(a) 8.58 kg m^2 s^{-2} (b) 10.86 kg m^2 s^{-2}

(c) 2.86 kg m^2 s^{-2} (d) 6.66 kg m^2 s^{-2}

137. Point masses m_1 and m_2 are placed at the opposite ends of a rigid rod of length L, and negligible mass. The rod is to be set rotating about an axis perpendicular to it. The position of point P on this rod through which the axis should pass so that the work required to set the rod rotating with angular velocity ω_0 is minimum, is given by



(a) $x = \frac{m_1}{m_2}L$ (b) $x = \frac{m_2}{m_1}L$

(c) $x = \frac{m_2 L}{m_1 + m_2}$ (d) $x = \frac{m_1 L}{m_1 + m_2}$

138. A force $F = \hat{i}\ \ 3\hat{j}\ \ 6\hat{k}$ is acting at a point $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$. The value of α for which angular momentum about origin is conserved is

(a) 2 (b) zero

(c) 1 (d) –1

139. A solid cylinder of mass 50 kg and radius 0.5 m is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of 2 revolutions s^{-2} is

(a) 25 N (b) 50 N

(c) 78.5 N (d) 157 N

140. A rod of weight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is

(a) $\frac{Wd}{x}$ (b) $\frac{W(d-x)}{x}$

(c) $\frac{W(d-x)}{d}$ (d) $\frac{Wx}{d}$

141. A uniform solid cylindrical roller of mass 'm' is being pulled on a horizontal surface with force F parallel to the surface and applied at its centre. If the acceleration of the cylinder is 'a' and it is rolling without slipping then the value of 'F' is

(a) ma (b) $\frac{5}{3}ma$

(c) $\frac{3}{2}ma$ (d) 2 ma

142. Consider a thin uniform square sheet made of a rigid material. If its side is '*a*' mass *m* and moment of inertia I about one of its diagonals, then

(a) $I > \frac{ma^2}{12}$ (b) $\frac{ma^2}{24} < I < \frac{ma^2}{12}$

(c) $I = \frac{ma^2}{24}$ (d) $I = \frac{ma^2}{12}$

143. A particle of mass 2 kg is on a smooth horizontal table and moves in a circular path of radius 0.6 m. The height of the table from the ground is 0.8 m. If the angular speed of the particle is 12 rad s^{-1}, the magnitude of its angular momentum about a point on the ground right under the centre of the circle is

(a) 14.4 kg $m^2 s^{-1}$ (b) 8.64 kg $m^2 s^{-1}$

(c) 20.16 kg $m^2 s^{-1}$ (d) 11.52 kg $m^2 s^{-1}$

144. A uniform thin rod AB of length L has linear mass density $\mu(x) = a + \frac{bx}{L}$, where x is measured from A. If the CM of the rod lies at a distance of $\left(\frac{7}{12}\right)L$ from A, then a and b are related as :

(a) $a = 2b$ (b) $2a = b$

(c) $a = b$ (d) $3a = 2b$

145. Two objects P and Q initially at rest move towards each other under mutual force of attraction. At the instant when the velocity of P is v and that of Q is 2v, the velocity of centre of mass of the system is

(a) v (b) 3v

(c) 2v (d) zero

146. A body rolls down an inclined plane. If its kinetic energy of rotation is 40% of its kinetic energy of translation motion, then the body is

(a) hollow cylinder (b) ring

(c) solid disc (d) solid sphere

147. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is

(a) $\frac{4MR^2}{9\sqrt{3}\pi}$ (b) $\frac{4MR^2}{3\sqrt{3}\pi}$

(c) $\frac{MR^2}{32\sqrt{2}\pi}$ (d) $\frac{MR^2}{16\sqrt{2}\pi}$

148. Two discs rotating about their respective axis of rotation with angular speeds 2 rads^{-1} and 5 rads^{-1} are brought into contact such that their axes of rotation coincide. Now, the angular speed of the system becomes 4 rads^{-1}. If the moment of inertia of the second disc is 1×10^{-3} kg m^2, then the moment of inertia of the first disc (in kg m^2) is

(a) 0.25×10^{-3} (b) 1.5×10^{-3}

(c) 1.25×10^{-3} (d) 0.5×10^{-3}

149. A wheel is rotating at 1800 rpm about its own axis. When the power is switched off, it comes to rest in 2 minutes. Then the angular retardation in rad s^{-1} is

(a) 2π (b) π

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

150. If the angular momentum of a particle of mass m rotating along a circular path of radius r with uniform speed is L, the centripetal force acting on the particle is

(a) $\frac{L^2}{mr^2}$ (b) $\frac{L^2}{mr}$

(c) $\frac{L}{mr}$ (d) $\frac{L^2 m}{r}$

151. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension:

(a) angular momentum is conserved.

(b) angular momentum changes in magnitude but not in direction.

(c) angular momentum changes in direction but not in magnitude.

(d) angular momentum changes both in direction and magnitude.

152. A solid cylinder of mass 2 kg and radius 0.1 m rolls down an inclined plane of height 3m without slipping. Its rotational kinetic energy when it reaches the foot of the plane would be :

(a) 22.7 J (b) 19.6 J

(c) 10.2 J (d) 9.8 J

153. A solid sphere and a hollow sphere of the same material and of a same size can be distinguished without weighing

(a) by determining their moments of inertia about their coaxial axes

(b) by rolling them simultaneously on an inclined plane

(c) by rotating them about a common axis of rotation

(d) by applying equal torque on them

154. A solid sphere rolls down two different inclined planes of same height, but of different inclinations. In both cases

(a) speed and time of descent will be same

(b) speed will be same, but time of descent will be different

(c) speed will be different, but time of descent will be same

(d) speed and time of descent both are different

155. The ratio of the accelerations for a solid sphere (mass 'm' and radius 'R') rolling down an incline of angle 'θ' without slipping and slipping down the incline without rolling is

(a) 5 : 7 (b) 2 : 3

(c) 2 : 5 (d) 7 : 5

FACT/DEFINITION TYPE QUESTIONS

1. (a)
2. (b) Depends on the distribution of mass in the body.
3. (d) By definition, position vector of centre of mass of two particle system is such that the product of total mass of the system and position vector of centre of mass is equal to the sum of products of masses of two particles and their respective position vectors i.e.

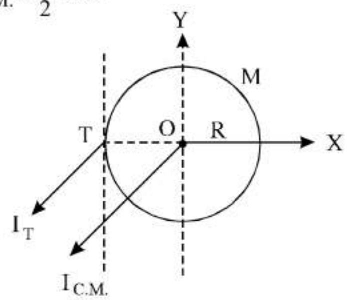
$$(m_1 + m_2)\vec{R} = m_1 \vec{r_2} + m_r \vec{r_2}$$

4. (a) 5. (d) 6. (d) 7. (a)
8. (c) 9. (a)
10. (d) The position of centre of mass of a system depends upon mass, relative distance, position and symmetry of the body.

$$R_{CM} = \frac{\sum m_i r_i}{\sum m_i}$$

11. (b) From $v = r\omega$, linear velocities (v) for particles at different distances (r) from the axis of rotation are different.
12. (a) Since torque $\vec{\tau}$ is rotational analogue of force $\vec{F}$ and $\vec{F}$ = mass × acceleration, therefore torque $\vec{\tau}$ = moment of inertia xangular acceleration ($I \propto$) as moment of inertia is rotational analogue of mass.
13. (c) In analogy to Newton's second law of motion in linear motion,
Force = rate of charge of linear momentum, in angular motion
Torque = rate of change of angular momentum
14. (b) Only the transverse component contributes to the torque.
15. (c) Turning moment of force = Force × distance from the axis of rotation.
Thus a small force is required to produce a given turning moment, when distance is large. That is why handle of a screw is made wider.
16. (a) $P = \tau\omega$. ($P = FV$ in translational motion)
Since $\tau = I\alpha$
$\therefore \quad P = I\omega\alpha$
17. (a) From $L = I\omega$, we find that angular momentum is directly proportional to the moment of inertia.
18. (b) A couple can produce rotational motion only and not linear motion.
19. (b) As $L = I\omega$ = constant, therefore, when I decreases, ω will increase.
20. (d) As angular momentum, $\vec{L} = \vec{r} \times \vec{p}$, therefore, direction of $\vec{L}$ is along a line perpendicular to the plane of rotation.
21. (b) The motion of a rigid body which is not pivoted or fixed in some way is either a pure **translation** or a combination of translation and rotation. The motion of a rigid body which is pivoted or fixed in some way is rotation.
22. (c) Planar, perpendicular and perpendicular. The moment of inertia of a planar body about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.
23. (b) By bending his body, he decreases his moment of inertia. This would increase his angular velocity.
24. (c) M.I. of a uniform circular disc of radius 'R' and mass 'M' about an axis passing through C.M. and normal to the disc is

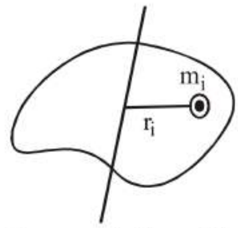
$$I_{C.M.} = \frac{1}{2} MR^2$$



From parallel axis theorem

$$I_T = I_{C.M.} + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

25. (d) Force in linear motion corresponds to torque in rotational motion.
26. (b) Angular momentum will remain conserved as no torque is exerted by the boy.
27. (c) In rotation of a rigid body about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis.
28. (c)
29. (c) Force changes the linear momentum of the body. The external force (or force) change the translational motion of the rigid body. But this is not the only effect of the force. The total torque due to the force on the body changes the rotational state of motion of the rigid body.
30. (a)
31. (a) Basic equation of moment of inertia is given by $I = \sum_{i=1}^{n} m_i r_i^2$



where m_i is the mass of i^{th} particle at a distance of r_i from axis of rotation.
Thus it does not depend on angular velocity.

32. (d) Moment of inertia of a hollow cylinder of mass M and radius r about its own axis is Mr^2.

33. (a)
(a) Moment of inertia of a ring about its axis and perpendicular to its plane = Mr^2
(b) Moment of inertia of disc about its axis and perpendicular to its plane = $\frac{1}{2}Mr^2 = 0.5\ Mr^2$
(c) Moment of inertia of a solid sphere about one of its diameter = $\frac{2}{5}Mr^2 = 0.4\ Mr^2$
(d) Moment of inertia of a spherical shell about one of its diameter = $\frac{2}{3}Mr^2 = 0.66\ Mr^2$

Therefore, the moment of inertia of the ring is highest

34. (a) Radius of gyration of a body depends on the axis of rotation.

35. (d)

36. (b) Moment of inertia is a tensor quantity.

37. (d) Ratio of M.I is

$$\frac{M_A r^2}{M_B (2r)^2} = \frac{I_A}{I_B}$$

$$\frac{I_A}{I_B} = \frac{1}{4} \quad [\because M_A = M_B]$$

$$\text{or,} \quad I_A = \frac{I_B}{4}$$

38. (a)

39. (b) Because the entire mass of a ring is at its periphery i.e. at maximum distance from the centre and $I = Mr^2$

40. (a)

41. (d) I_{xy}, moment of inertia of a ring about its tangent in the plane of ring $I_{x^1y} = \frac{3}{2}MR^2$

Moment of inertia about a tangent perpendicular to the plane of ring $I_{xy} = 2MR^2$

$$\therefore \quad I_{xy} = \frac{3}{4}(2MR^2) = \frac{3}{2}MR^2$$

$$\text{or} \quad I_{xy} = \frac{3}{4} I_{x^1y^1}$$

42. (d) From $I = MR^2$, moment of inertia depends on the mass and size of the body. It also depends on the distribution of mass, thus it depends on the shape of the body as well.

43. (d)

44. (c) A rigid body is in mechanical equilibrium if
(i) it is in translational equilibrium and
(ii) it is in rotational equilibrium.

45. (b) Torque $\vec{\tau} = \vec{r} \times \vec{F}$

46. (d) $K.E. = \frac{1}{2}mV^2$

Since $V = \omega r$

$$\therefore \quad K.E. = \frac{1}{2}mV^2 = \frac{1}{2}m\omega^2 r^2$$

$$= \frac{1}{2}I\omega^2 \qquad (\because mr^2 = I)$$

47. (a) Angular momentum of particle is given by :

$L = I\omega = mr^2\omega = 2\pi mr^2 f \qquad [\because W = 2\pi f]$

If frequency is halved then,

$$L' = I\omega' = mr^2\frac{\omega}{2} = \pi mr^2 f$$

$$\therefore L' = \frac{L}{2}$$

48. (a) Moment of inertia of ring is greater than disc.
About a diameter

$$I_{ring} = \frac{1}{2}MR^2$$

$$\text{and} \quad I_{disc} = \frac{1}{4}MR^2$$

49. (b) We know that $\tau_{ext} = \frac{dL}{dt}$

if angular momentum is conserved, it means change in angular momentum = 0

or, $dL = 0$

$$\frac{dL}{dt} = 0 \Rightarrow \tau_{ext} = 0$$

Thus total external torque = 0.

50. (d) We know that $\vec{\tau_c} = \frac{d\vec{L_c}}{dt}$

where $\vec{\tau_c}$ Torque about the center of mass of the body

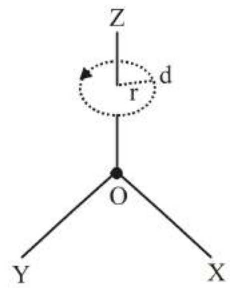
and $\vec{L_c}$ = Angular momentum about the center of mass of the body. Central forces act along the center of mass. Therefore torque about center of mass is zero.

When $\vec{\tau_c} = 0$ then $\vec{L_c}$ = constt.

51. (b) P.E. of the body is converted into both translational KE and rotational K.E i.e., $P.E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

52. (d)

53. (d) A rigid body performs a pure rotational motion of every particle of the body moves in a circle and the centres of all the circles lie on a straight line called the axis of rotation. Particles lying on the axis of rotation remain stationary. Hence, motion of fan moving around the axis satisfies this criteria is purely rotational.



54. (c) Frictional force acts upwards forming a couple and hence creating torque on the body which increases the angular velocity of the body.

55. (a) Kinetic energy of a rolling body $k = \frac{1}{2}mv^2\left(1+\frac{k}{R^2}\right)$.

According to law of conservation of energy, potential energy lost by rolling body is equal to the final kinetic energy of the body.

$$\therefore \quad \cancel{m}gh = \frac{1}{2}\cancel{m}v^2\left(1+\frac{k^2}{R^2}\right)$$

$$\therefore \quad v^2 = \frac{2gh}{\left(1+\frac{k^2}{R^2}\right)}$$

Thus it is independent of the mass of the bodies.

For ring, $k^2 = R^2$

$\therefore \quad V = \sqrt{gh}$

For cylinder, $k^2 = R^2/2$

$\therefore \quad V = \sqrt{\frac{4gh}{3}}$

For solid sphere, $k^2 = 2R^2/5$

$\therefore \quad V = \sqrt{\frac{10gh}{7}}$

Thus sphere has greatest and ring has least velocity of centre of mass at the bottom of the inclined plane.

STATEMENT TYPE QUESTIONS

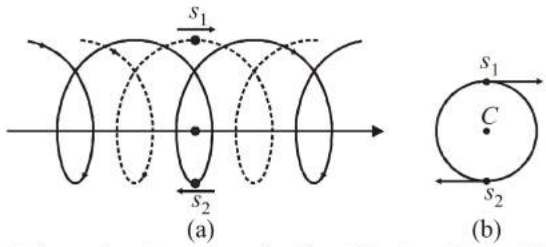
56. (d)
57. (a) Centre of mass of a body can coincide with its geometrical centre when the body has a uniform mass density.
58. (c) Centre of mass does not necessarily lie only where there is mass. It can lie outside the body as well. For e.g. Centre of mass of circular ring lies in the centre of the ring where there is no mass.
59. (c) Linear velocity has magnitude and direction both therefore it is a vector quantity. Angular velocity has a fixed direction when a body rotates about a fixed axis.
60. (d) Centre of mass depends on the distribution of mass.
61. (c)
I. A body in translatory motion shall have angular momentum unless the fixed point about which angular momentum is taken lies on the line of motion of the body.
II. If $\vec{A}$ points vertically upwards and $\vec{B}$ points towards east then $\vec{A} \times \vec{B}$ points towards north.
62. (b) Moment of inertia is a tensor quantity.
$\tau = I\alpha \qquad \therefore$ for constant I
α can change only if torque acts.
63. (b) If Earth shrinks suddenly, its radius R would decrease and $I = \frac{2}{5}MR^2$ would decrease. Thus, ω increases to keep angular momentum constant. Hence the length of the day will decrease.

MATCHING TYPE QUESTIONS

64. (a) (A)→(2); (B)→(3); C→(1); (D)→(4)
65. (c) (A)→(2); (B)→(1); C→(4); (D)→(3)
Rolling motion → combination of translatory and rotatory motion
Rate of change of angular momentum → torque
$\frac{dL}{dt} = \tau$
Moment of inertia of a hollow cylinder about axis $= MR^2$
Theorem of parallel axis $I_Z^1 = I_Z + Ma^2$
66. (a) (A)→(1); (B)→(6); C→(5); (D)→(4); (E)→(2)
For translational equilibrium, total ext force = 0
$\Sigma F = 0$
Moment of inertia of disc $= \frac{MR^2}{2}$
For rotational equilibrium, net external torque = 0
$\Sigma\tau = 0$
Kinetic energy of rolling body = K. E. of translation + K. E. of rotation.
$K = \frac{1}{2}mVcm^2 + \frac{1}{2}I\omega^2$
Moment of inertia of a ring $= MR^2$
67. (d) (A)→(1); (B)→(4); C→(3); (D)→(2)
68. (c) (A)→(4); (B)→(3); C→(2); (D)→(5)
69. (d) (A)→(2); (B)→(4); C→(5); (D)→(3)
70. (a) (A)→(4); (B)→(3); C→(2); (D)→(1)

DIAGRAM BASED QUESTIONS

71. (a) When no external force acts on the binary star, its CM will move like a free particle [Fig. (a)]. From the CM frame, the two stars will seems to move in a circle about the CM with diametrically opposite positions.



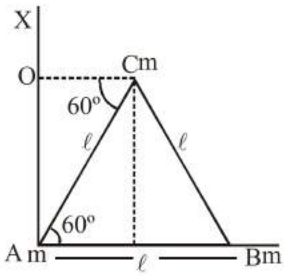
(a) Trajectories of two stars. S_1 (dotted line) and S_2 (solid line) forming a binary system with their centre of mass C in uniform motion
(b) The same binary system, with the centre of mass C at rest.

So, to understand the motion of a complicated system, we can separate the motion of the system into two parts. So, the combination of the motion of the CM and motion about the CM could described the motion of the system.

72. (a) According to parallel axis theorem of the moment of Inertia
$I = I_{cm} + md^2$
d is maximum for point B so I_{max} about B.

73. **(b)**

74. **(d)** $I_{AX} = m(AB)^2 + m(OC)^2 = m\ell^2 + m(\ell\cos 60^o)^2$
$= m\ell^2 + m\ell^2/4 = 5/4\, m\ell^2$

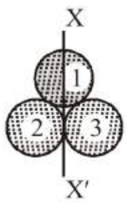


75. **(c)** Moment of inertia of shell 1 along diameter

$I_{diameter} = \frac{2}{3}MR^2$

Moment of inertia of shell 2 = m. i of shell 3

$= I_{tengential} = \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$



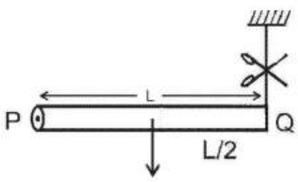
So, I of the system along x x[1]
$= I_{diameter} + (I_{tengential}) \times 2$

or, $I_{total} = \frac{2}{3}MR^2 + \left(\frac{5}{3}MR^2\right) \times 2$

$= \frac{12}{3}MR^2 = 4MR^2$

76. **(b)** The distribution of mass about axis EF is minimum so radius of gyration is minimum and therefore moment of inertia is minimum about EF.

77. **(d)**



Weight of the rod will produce the torque

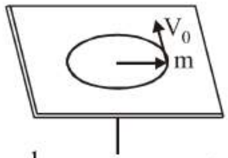
$\tau = mg\frac{L}{2} = I\alpha = \frac{mL^2}{3}\alpha \quad \left[\because I_{rod} = \frac{ML^2}{3}\right]$

Hence, angular acceleration $\alpha = \frac{3g}{2L}$

78. **(b)** Applying angular momentum conservation

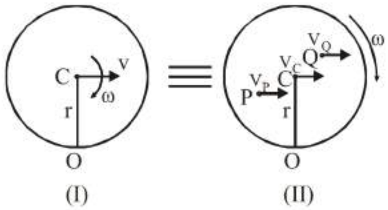
$mV_0R_0 = (m)(V^1)\left(\frac{R_0}{2}\right)$

$\therefore \quad v^1 = 2V_0$



Therefore, new KE $= \frac{1}{2}m(2V_0)^2 = 2mv_0^2$

79. **(a)**

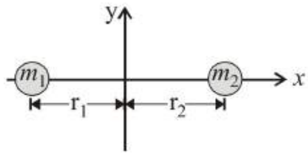


From Fig. (I), we have OC = r (radius)
Therefore, $v = r\omega$
Since, ω = constant, therefore $v \propto r$
Now, form Fig (II), it is clear that the distance,
$OP < OC < OQ \Rightarrow V_P < V_C < V_Q$ or $V_Q > V_C > V_P$.

ASSERTION- REASON TYPE QUESTIONS

80. **(b)** **81.** **(c)**

82. **(a)** If centre of mass of system lies at origin then $\vec{r}_{cm} = 0$



$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$

$\therefore \quad m_1\vec{r}_1 + m_2\vec{r}_2 = 0$

or $m_1r_1 = m_2r_2$

83. **(b)**

84. **(b)** External force on the system $F_{ext} = M\frac{d}{dt}(\vec{v}_{cm} \quad)$

If system is isolated i.e. $F_{ext} = 0$ then $\vec{v}_{cm}$ constant

If initially the velocity of centre of mass is zero then it will remain zero.

85. **(a)** As the concept of centre of mass is only theoretical, therefore in practice no mass may lie at the centre of mass. For example, centre of mass of a uniform circular ring is at the centre of the ring where there is no mass.

86. **(c)** The position of centre of mass of a body depends on shape, size and distribution of mass of the body. The centre of mass does not lie necessarily at the centre of the body.

87. (a)

88. (d) When particle moves with constant velocity $\vec{v}$ then its linear momentum has some finite value $(\vec{P} = m\vec{v})$.
Angular momentum (L) = Linear momentum (P) × Perpendicular distance of line of action of linear momentum form the point of rotation (d)
So if $d \neq 0$ then $L \neq 0$, but if d = 0 then L may be zero. So we can conclude that angular momentum of a particle moving with constant velocity is not always zero.

89. (d) The earth is not slowing down. The angular momentum of the earth – moon system is conserved.

90. (a) Both the assertion and reason are true.
For central forces,

$$\tau = \frac{dL}{dt} = 0$$

∴ L = constant

91. (c) Torque = Force × perpendicular distance of line of action of force from the axis of rotation (d).
Hence for a given applied force, torque or true tendency of rotation will be high for large value of d. If distance d is smaller, then greater force is required to cause the same torque, hence it is harder to open or shut down the door by applying a force near the hinge.

92. (b) $\vec{\tau} = \frac{d\vec{L}}{dt}$ and $L = I\omega$

93. (a) Torque $\tau = \frac{dL}{dt}$ ∴ $\tau = 0$, L = constant.
$I\omega$ = constant
∴ ω is constants as long as I is constant.

94. (d) Torque is a vector whose direction is perpendicular to F since $\vec{\tau} = \vec{r} \times \vec{F}$.

95. (d) The moment of inertia of a rigid body reduces to its minimum value, when the axis of rotation passes through its centre of gravity because the weight of a rigid body always acts through its centre of gravity.

96. (c) An ice-skater stretches out arms and legs during performance to take advantage of principle of conservation of angular momentum. As on doing so, their moment of inertia increases or decreases respectively and hence the angular velocity of spin motion decreases or increases accordingly.

97. (c) As the polar ice melts, water so formed flows towards the equator. The moment of inertia of the earth increases. To conserve angular momentum, angular velocity decreases. This increases the length ($T = 2\pi/\omega$) of the day.

98. (d) The moment of inertia of a particle about an axis of rotation is given by the product of the mass of the particle and the square of the perpendicular distance of the particle from the axis of rotation. For different axis, distance would be different, therefore moment of inertia of a particle changes with the change in axis of rotation.

99. (d) Radius of gyration of body is not a constant quantity. Its value changes with the change in location of the axis of rotation. Radius of gyration of a body about a given axis is given as

$$K = \sqrt{\frac{r_1^2 + r_2^2 + ...r_n^2}{n}}$$

100. (d) For a disc rolling without slipping on a horizontal rough surface with uniform angular velocity, the acceleration of lowest point of disc is directed vertically upwards and is not zero (Due to translation part of rolling, acceleration of lowest point is zero. Due to rotational part of rolling, the tangential acceleration of lowest point is zero and centripetal acceleration is non-zero and upwards). Hence assertion is false.

101. (b)

102. (b)

103. (a) In sliding down, the entire potential energy is converted into kinetic energy. While in rolling down some part of potential energy is converted into K.E. of rotation. Therefore linear velocity acquired is less.

104. (c) $K_N = K_R + K_T$
This equation is correct for any body which is rolling with slipping
For the ring and hollow cylinder only $K_R = K_T$ i.e. $K_N = 2K_T$

CRITICALTHINKING TYPE QUESTIONS

105. (c) 106. (d) 107. (a) 108. (c) 109. (a)

110. (c) $m_1 d = m_2 d_2 \Rightarrow d_2 = \frac{m_1 d}{m_2}$

111. (a) $X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$$X_{cm} = \frac{300 \times (0) + 500(40) + 400 \times 70}{300 + 500 + 400}$$

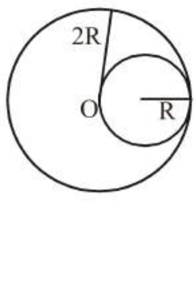
$$X_{cm} = \frac{500 \times 40 + 400 \times 70}{1200}$$

$$X_{cm} = \frac{50 + 70}{3} = \frac{120}{3} = 40\,\text{cm}$$

112. (a) Does not shift as no external force acts. The centre of mass of the system continues its original path. It is only the internal forces which comes into play while breaking.

113. (b) Let the mass per unit area be σ.
Then the mass of the complete disc

$$= \sigma[\pi(2R)^2] = 4\pi\sigma R^2$$



The mass of the removed disc $= \sigma(\pi R^2) = \pi\sigma R^2$

Let us consider the above situation to be a complete disc of radius 2R on which a disc of radius R of negative mass is superimposed. Let O be the origin. Then the above figure can be redrawn keeping in mind the concept of centre of mass as :

$4\pi\sigma R^2 \xleftrightarrow{R}$

O $\quad \pi\sigma R^2$

$$X_{c.m} = \frac{(4\pi\sigma R^2)\times 0 + (-\pi\sigma R^2)R}{4\pi\sigma R^2 - \pi\sigma R^2}$$

$$\therefore \; x_{c.m} = \frac{-\pi\sigma R^2 \times R}{3\pi\sigma R^2}$$

$$\therefore \; x_{c.m} = -\frac{R}{3}$$

$$\Rightarrow \alpha = \frac{1}{3}$$

114. (d) $X_{CM} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$$= \frac{1\times 0 + 1\times 3 + 1\times 0}{1+1+1} = 1$$

$$Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times 0 + 1\times 0 + 1\times 4}{1+1+1} = \frac{4}{3}$$

Therefore the coordinates of centre of mass are $\left(1, \frac{4}{3}\right)$.

115. (a) When angular acceleration (α) is zero then torque on the wheel becomes zero.

$\theta(t) = 2t^3 - 6t^2$

$$\Rightarrow \frac{d\theta}{dt} = 6t^2 - 12t$$

$$\Rightarrow \alpha = \frac{d^2\theta}{dt^2} = 12t - 12 = 0$$

$\therefore t = 1$ sec.

116. (a) Tube may be treated as a particle of mass M at distance L/2 from one end.

Centripetal force $= Mr\omega^2 = \dfrac{ML}{2}\omega^2$

117. (b) Since no external torque is applied on planet + sun system, hence the angular momentum of the planet about the sun is constant.

since $L = mrv$

(i) At the nearest point, r is minimum, so v is maximum to conserve L.

(ii) At the farthest point, r is maximum, so v is minimum to conserve L.

118. (a) As mass decreases, moment of inertia I decreases. Since $L = I\omega$ is constant, therefore ω increases.

119. (a) When sand is poured on a rotating disc, the effective mass of the disc increases.

According to the law of conservation of angular momentum

$L = I\omega$ = constant

As the mass of the disc increases, its moment of inertia increases and therefore, its angular velocity decreases.

120. (b)

121. (d) When person suddenly draw his hands and weights towards his chest, the moment of inertia ($I = mr^2$) of the person decreases.

According to law of conservation of angular momentum if no external torque acts on a system, then the angular momentum of the system remains conserved, *ie*,

$L = I\omega$ = constant

or $\; I_1\omega_1 = I_2\omega_2$

As the moment of inertia decreases, the angular velocity will increase and therefore, person will rotate faster.

122. (b) A raw egg behaves like a spherical shell and a half boiled egg behaves like a solid sphere

$$\therefore \; \frac{I_r}{I_s} = \frac{2/3\, mr^2}{2/5\, mr^2} = \frac{5}{3} > 1$$

123. (b) Since no external torque act on gymnast, so angular momentum ($L = I\omega$) is conserved. After pulling her arms & legs, the angular velocity increases but moment of inertia of gymnast, decreases in, such a way that angular momentum remains constant.

124. (c) In a hollow sphere, the mass is distributed away from the axis of rotation. So, its moment of inertia is greater than that of a solid sphere.

125. (d) It decreases his moment of inertia and increases his angular velocity.

126. (b) Because radius of the sphere will be very less in comparison to ring (although mass is equal).

127. (d) $\tau \times \Delta t = L_0 \qquad \{\because \text{ since } L_f = 0\}$

$\Rightarrow \tau \times \Delta t = I\omega$

or $\tau \times 60 = 2 \times 2 \times 60\pi/60$

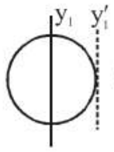
$\left(\because \; f = 60\text{rpm} \; \therefore \; \omega = 2\pi f = 2\pi \times \frac{60}{60}\right)$

$$\tau = \frac{\pi}{15}\,N-m$$

128. (d) Angular momentum will be conserved

$$I_1\omega = I_1\omega' + I_2\omega' \Rightarrow \omega' = \frac{I_1\omega}{I_1 + I_2}$$

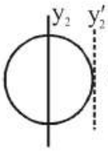
129. (d)



Circular disc (1)

$I_{y_1} = \frac{MR^2}{4}$

$\therefore I'_{y_1} = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$



Circular ring (2)

$I_{y_2} = \frac{MR^2}{2}$

$\therefore I'_{y_2} = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$

$I'_{y_1} = MK_1^2,\ I'_{y_2} = MK_2^2$

$\therefore \frac{K_1^2}{K_2^2} = \frac{I'_{y_1}}{I'_{y_2}} \Rightarrow K_1 : K_2 = \sqrt{5} : \sqrt{6}$

130. (d) $K = \frac{L^2}{2I} \Rightarrow L^2 = 2KI \Rightarrow L = \sqrt{2KI}$

$\frac{L_1}{L_2} = \sqrt{\frac{K_1}{K_2} \cdot \frac{I_1}{I_2}} = \sqrt{\frac{K}{K} \cdot \frac{I}{2I}} = \frac{1}{\sqrt{2}}$

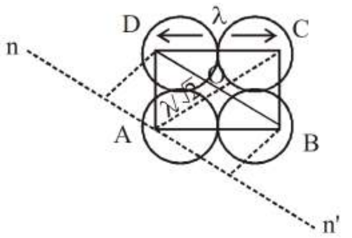
$L_1 : L_2 = 1 : \sqrt{2}$

131. (b) By theorem of parallel axes,

$I = I_{cm} + Md^2$

$I = I_0 + M(L/2)^2 = I_0 + ML^2/4$

132. (c)

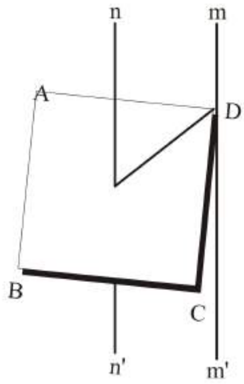


$I_{nn'}$ = M.I due to the point mass at B + M.I due to the point mass at D + M.I due to the point mass at C.

$I_{nn'} = 2 \times m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m(\sqrt{2}\ell)^2$

$= m\ell^2 + 2m\ell^2 = 3m\ell^2$

133. (d) $I_{nn'} = \frac{1}{12}M(a^2 + a^2) = \frac{Ma^2}{6}$



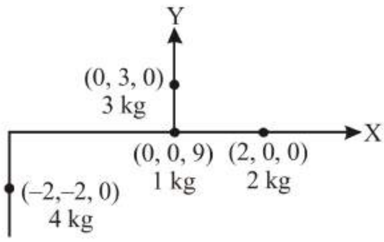
Also, $DO = \frac{DB}{2} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

According to parallel axis theorem

$I_{mm'} = I_{nn'} + M\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{Ma^2}{6} + \frac{Ma^2}{2}$

$= \frac{Ma^2 + 3Ma^2}{6} = \frac{2}{3}Ma^2$

134. (a) Moment of inertia of the whole system about the axis of rotation will be equal to the sum of the moments of inertia of all the particles.



$\therefore\ I = I_1 + I_2 + I_3 + I_4$

$= 0 + 0 + 27 + 16 = 43\ kg\,m^2$

135. (d) For circular disc 1

mass = M, radius $R_1 = R$

moment of inertia $I_1 = I_0$

For circular disc 2, of same thickness *t*,

mass = M, density $= \frac{\rho}{2}$

then $\pi R_2^2 t \times \frac{\rho}{2} = \pi R_1^2 t \times \rho = M$

$R_2^2 = 2R_1^2$

$R_2 = \sqrt{2}R_1 = \sqrt{2}R$

As we know, moment of inertia $I \propto (\text{Radius})^2$

$\therefore \frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^2$

$\frac{I_0}{I_2} = \left(\frac{R}{\sqrt{2}R}\right)^2 \Rightarrow I_2 = 2I_0$

136. (d) Given : speed $V = 54\ kmh^{-1} = 15\ ms^{-1}$

Moment of inertia, $I = 3\ kgm^2$

Time t = 15s

$\omega_i = \frac{V}{r} = \frac{15}{0.45} = \frac{100}{3}$ $\omega_f = 0$

$\omega_f = \omega_i + \alpha t$

$0 = \frac{100}{3} + (-\alpha)(15) \quad \Rightarrow \alpha = \frac{100}{45}$

Average torque transmitted by brakes to the wheel

$\tau = (I)(\alpha) = 3 \times \frac{100}{45} = 6.66 \text{ kgm}^2\text{s}^{-2}$

137. (c) Work required to set the rod rotating with angular velocity ω_0

$K.E. = \frac{1}{2} I\omega^2$

Work is minimum when I is minimum.
I is minimum about the centre of mass
So, $(m_1)(x) = (m_2)(L - x)$
or, $m_1 x = m_2 L - m_2 x$

$\therefore x = \frac{m_2 L}{m_1 + m_2}$

138. (d) From Newton's second law for rotational motion,

$\vec{\tau} = \frac{d\vec{L}}{dt}$, if $\vec{L}$ = constant then $\vec{\tau} = 0$

So, $\vec{\tau} = \vec{r} \times \vec{F} = 0$

$(2\hat{i} - 6\hat{j} - 12\hat{k}) \times (\alpha\hat{i} + 3\hat{j} + 6\hat{k}) = 0$

Solving we get $\alpha = -1$

139. (d) Here $\alpha = 2$ revolutions/s^2 = 4π rad/s^2 (given)

$I_{cylinder} = \frac{1}{2}MR^2 = \frac{1}{2}(50)(0.5)^2$

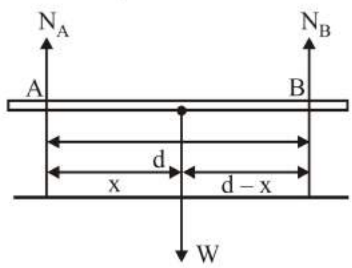
$= \frac{25}{4}$ Kg-m^2

As $\tau = I\alpha$ so $TR = I\alpha$

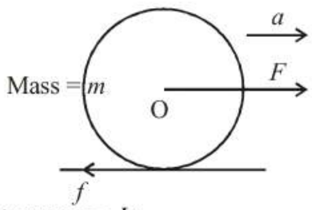
$\Rightarrow T = \frac{I\alpha}{R} = \frac{\left(\frac{25}{4}\right)(4\pi)}{(0.5)} N = 50\pi N = 157 N$

140. (c) By torque balancing about B
$N_A(d) = W(d - x)$

$N_A = \frac{W(d-x)}{d}$



141. (c) From figure,
$ma = F - f$ ....(i)



And, torque $\tau = I\alpha$

$\frac{mR^2}{2}\alpha = fR$

$\frac{mR^2}{2}\frac{a}{R} = fR \quad \left[\because \alpha = \frac{a}{R}\right]$

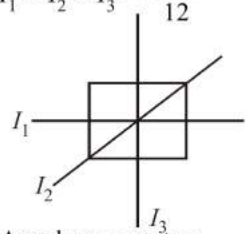
$\frac{ma}{2} = f$ ...(ii)

Put this value in equation (i),

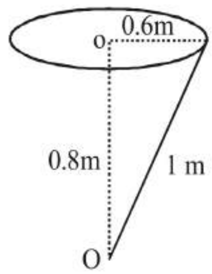
$ma = F - \frac{ma}{2}$ or $F = \frac{3ma}{2}$

142. (d) For a thin uniform square sheet

$I_1 = I_2 = I_3 = \frac{ma^2}{12}$



143. (a) Angular momentum,
L_0 = mvr sin 90°
= 2 × 0.6 × 12 × 1 × 1
[As $V = r\omega$, Sin 90° = 1]
So, L_0 = 14.4 kgm^2/s



144. (b) Centre of mass of the rod is given by:

$$x_{cm} = \frac{\int_0^L \left(ax + \frac{bx^2}{L}\right)dx}{\int_0^L \left(a + \frac{bx}{L}\right)dx}$$

$$= \frac{\frac{aL^2}{2} + \frac{bL^2}{3}}{aL + \frac{bL}{2}} = \frac{L\left(\frac{a}{2} + \frac{b}{3}\right)}{a + \frac{b}{2}}$$

Now $\frac{7L}{12} = \frac{\frac{a}{2}+\frac{b}{3}}{a+\frac{b}{2}}$

On solving we get, $b = 2a$

145. (d) Centre of mass is at rest

$\therefore V_{CM} = 0$

146. (d) $\frac{KE_{rot}}{KE_{trans}} = \frac{\frac{1}{2}mv^2\left(\frac{k^2}{R^2}\right)}{\frac{1}{2}mv^2} = \frac{K^2}{R^2} = 0.4 = \frac{2}{5}$

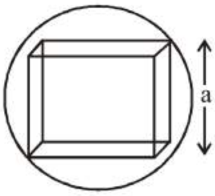
$\Rightarrow \quad k^2 = \frac{2}{5}R^2$

Therefore it is a solid sphere.

147. (a) Here $a = \frac{2}{\sqrt{3}}R$

Now, $\frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{a^3}$

$= \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3} = \frac{\sqrt{3}}{2}\pi.$



$M' = \frac{2M}{\sqrt{3}\pi}$

Moment of inertia of the cube about the given axis,

$I = \frac{M'a^2}{6}$

$= \frac{\frac{2M}{\sqrt{3}\pi}\times\left(\frac{2}{\sqrt{3}}R\right)^2}{6} = \frac{4MR^2}{9\sqrt{3}\pi}$

148. (d) According to law of conservation of angular momentum,

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

Substituting the values of $\omega_1 = 2$ rad s^{-1}

$\omega_2 = 5$ rad s^{-1}

$I_2 = I\times 10^{-3}$ kg m^2

$I_1\times 2 + 1\times 10^{-3}, \times 5 = (I_1 + 1\times 10^{-3})\times 4$

$\Rightarrow 2I_1 + 5\times 10^{-3} = 4I_1 + 4\times 10^{-3}$

$\Rightarrow 2I_1 = 1\times 10^{-3}$

$\Rightarrow I_1 = \frac{1\times 10^{-3}}{2} = 0.5\times 10^{-3}$ kg m^2

149. (c) Given, no. of rotation $n = 1800$ rpm $= 1800$ rps

Time, $t = 2$ minutes $= 120$s

$\therefore$ Initial angular speed $\omega_0 = \frac{2\pi\times 1800}{60}$ rad s^{-1}

$= 60\pi$ rad s^{-1}

Final angular speed (as wheel comes to rest)

$\omega = 0$

$\therefore$ Angular retardation $= \frac{\omega_0 - \omega}{t}$

$= \frac{60\pi - 0}{120} = \frac{\pi}{2}$ rad s^{-2}

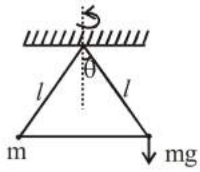
150. (a) From the relation, angular momentum, $L = mvr$

$v = \frac{L}{mr}$

$\therefore$ Centripetal force acting on the particle

$F = \frac{mv^2}{r} = \frac{m\left(\frac{L}{mr}\right)^2}{r} = \frac{L^2}{mr^3}$

151. (c) Torque working on the bob of mass m is, $\tau = mg\times \ell\sin\theta$. (Direction parallel to plane of rotation of particle)



As τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same.

152. (b) $mgh = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{MR^2}{2}\right)\left(\frac{V}{R}\right)^2$

$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$

Rotation K.E. $= \frac{1}{4}mv^2 = \frac{1}{4}\times\frac{4}{3}mgh$

$= \frac{mgh}{3} = 2\times 9.8\times(3/3) = 19.6$J

153. (b) Time of descent will be less for solid sphere. i.e., solid, sphere will reach first at the bottom of inclined plane.

154. (b) In pure rolling, mechanical energy remains conserved. Therefore, when heights of inclines are equal, speed of sphere will be same in both the case. But as acceleration down the plane, $a \propto \sin\theta$. Therefore, acceleration and time of descent will be different.

155. (a) For solid sphere rolling without slipping on inclined plane, acceleration

$a_1 = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$

For solid sphere slipping on inclined plane without rolling, acceleration

$a_2 = g\sin\theta$

Therefore required ratio $= \frac{a_1}{a_2}$

$= \frac{1}{1+\frac{K^2}{R^2}} = \frac{1}{1+\frac{2}{5}} = \frac{5}{7}$